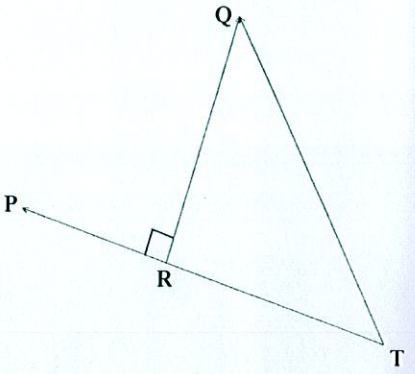
Let Q be the point $(-9, 7, 6)$, T be the point $(-1, 5, -4)$, and $\overrightarrow{TP}$ be the vector $-6\vec{j}+9\vec{k}$ in the diagram on the right.

SCORE: ____ / 80 PTS



NOTE: The diagram is NOT drawn to scale.

[a] Find the coordinates of P.

(6)

$\langle 0,-6,9\rangle = \langle x-^{-}1, y-5, z-^{-}4\rangle$

(3)
$x+1=0$
$y-5=-6$
$z+4=9$

$(x,y,z) = (-1,-1,5)$ (3)

[b] Find $\overrightarrow{RQ}$. Write your final answer as a linear combination of $\vec{i}$, $\vec{j}$ and $\vec{k}$.

(24)

$\overrightarrow{TQ} = \langle -9-^{-}1, 7-5, 6-^{-}4\rangle = \langle -8,2,10\rangle$ (4)

$\overrightarrow{TR} = \text{PROJ}_{\overrightarrow{TP}}\overrightarrow{TQ} = \dfrac{\langle -8,2,10\rangle \cdot \langle 0,-6,9\rangle}{\langle 0,-6,9\rangle \cdot \langle 0,-6,9\rangle}\langle 0,-6,9\rangle$

(4) $= \dfrac{78}{117}\langle 0,-6,9\rangle$

(4) $= \frac{2}{3}\langle 0,-6,9\rangle = \langle 0,-4,6\rangle$ (4)

$\overrightarrow{RQ} = \overrightarrow{TQ} - \overrightarrow{TR} = \langle -8-0, 2-^{-}4, 10-6\rangle$ (4)

$= -8\vec{i}+6\vec{j}+4\vec{k}$ (4)

[c] Find parametric equations of the line passing through T that is perpendicular to both $\overrightarrow{TP}$ and $\overrightarrow{TQ}$.

(14)

$\overrightarrow{TP} \times \overrightarrow{TQ} = \langle 0,-6,9\rangle \times \langle -8,2,10\rangle$ (4)

$= \langle -60-18, -(0-^{-}72), 0-48\rangle$

$= \langle -78,-72,-48\rangle = -6\langle 13,12,8\rangle$ (4)

(3) (3)
$x = -1 + 13t$
$y = 5 + 12t$
$z = -4 + 8t$

[d] Starting at point T, you move 8 units to the left, 6 units backwards and 3 units up. What are the coordinates of your ending point, and in which octant are you ?

(10)

(3) $(-1-6, 5-8, -4+3) = (-7,-3,-1)$ (3)

$O_{3+4} = O_7$ (4)

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$x = 6$

[e] Find symmetric equations of the line passing through Q and parallel to $y = 7t + 3$.

$z = 4 - 8t$

12

$\vec{d} = \langle 0, 7, -8 \rangle$

(4) $x = -9$, (4) $\frac{y-7}{7} = \frac{z-6}{-8}$ or (4) $\frac{6-z}{8}$

[f] Find the general form ($Ax + By + Cz + D = 0$) of the equation of the plane passing through P, Q and T.

8

$\vec{n} = \langle 13, 12, 8 \rangle$ FROM [c] (4)

$13(x - -1) + 12(y - 5) + 8(z - -4) = 0$

$13x + 12y + 8z - 15 = 0$ (4)

[g] A force represented by $\overrightarrow{TP}$ is applied to an object as it moves from T to Q.

Find the work done, if the force is measured in pounds, and all coordinates are in feet.

6

$\overrightarrow{TP} \cdot \overrightarrow{TQ} = 78$ FOOT-POUNDS (2)

(4) FROM [b]

If $\vec{u} = < 4, 6, 9 >$ is perpendicular to $\vec{p} = < a, -3, -2 >$, and also parallel to $\vec{w} = < a, b, c >$, find the values of a, b and c.

SCORE: ____ / 15 PTS

$\vec{u} \cdot \vec{p} = 0 \rightarrow 4a - 18 - 18 = 0$ (3)

$4a = 36$

$a = 9$ (3)

$\langle 9, b, c \rangle = k \langle 4, 6, 9 \rangle$

(3) $9 = 4k \rightarrow k = \frac{9}{4}$

$b = 6k \rightarrow b = \frac{27}{2}$ (3)

$c = 9k \rightarrow c = \frac{81}{4}$ (3)

Two forces are applied to an object. **SCORE: ____ / 55 PTS**

The first force is represented by the vector $\vec{F_1}$ with magnitude 8 and direction angle 102°.

The second force is represented by the vector $\vec{F_2}$.

[a] Find the component form of $\vec{F_1}$. Round all final answers to 1 decimal place. (10)

$\langle 8\cos 102^\circ, 8\sin 102^\circ \rangle = \langle -1.7, 7.8 \rangle$ (5) (5)

[b] If the resultant of the two forces is $\vec{F} = \langle -6, 10 \rangle$, find the component form of $\vec{F_2}$. Round all final answers to 1 decimal place. (10)

$\vec{F} = \vec{F_1} + \vec{F_2}$

$\vec{F_2} = \vec{F} - \vec{F_1}$

$= \langle -6 - -1.7, 10 - 7.8 \rangle$ (5)

$= \langle -4.3, 2.2 \rangle$ (5)

[c] Find the magnitude and direction angle (in degrees) of $\vec{F_2}$. Round all final answers to 1 decimal place. (15)

$\|\vec{F_2}\| = \sqrt{(-4.3)^2 + (2.2)^2} = 4.8$ (5)

$\theta_{\vec{F_2}} = 180^\circ + \tan^{-1}\frac{2.2}{-4.3} = 152.9^\circ$ (5) (5)

[d] Find a vector of magnitude 8 perpendicular to $\vec{F}$ (the resultant mentioned in [b]). (20)

NO DECIMAL APPROXIMATIONS ALLOWED.

$\langle -6, 10 \rangle \cdot \langle a, b \rangle = 0$

$-6a + 10b = 0$

LET $a = 5, b = 3$

$\frac{8}{\|\langle 5, 3 \rangle\|}\langle 5, 3 \rangle$ (5) (5)

$= \frac{8}{\sqrt{34}}\langle 5, 3 \rangle$ (5)

$= \left\langle \frac{40}{\sqrt{34}}, \frac{24}{\sqrt{34}} \right\rangle$

$= \left\langle \frac{20\sqrt{34}}{17}, \frac{12\sqrt{34}}{17} \right\rangle$ (5)